PROCEEDINGS

OF

THE ROYAL SOCIETY.

March 3, 1892.

Mr. JOHN EVANS, D.C.L., LL.D., Treasurer, in the Chair.

A List of the Presents received was laid on the table, and thanks ordered for them.

In pursuance of the Statutes, the names of the Candidates for election into the Society were announced, as follows:—

Armstrong, Robert Young, Lieut.-Col. Beddard, Frank Evers, M.A. Beever, Charles Edward, M.D. Blake, Rev. John Frederick, M.A. Boulenger, George Albert. Brennand, William. Buzzard, Thomas, M.D. Callendar, Hugh Longbourne. Davis, James William, F.G.S. Dibdin, W. J., F.C.S. Dreschfeld, Professor Julius, M.D. Dresser, Henry Eales, F.L.S. Dunstan, Professor Wyndham R. Eaton, Rev. Alfred Edwin, M.A. Ellis, William, F.R.A.S. Etheridge, Robert, F.G.S. Ewart, Professor J. Cossar, M.D. Fleming, Professor John Ambrose, M.A. Foster, Professor Clement $_{
m Le}$ Neve, D.Sc. Gadow, Hans, M.A.

Giffen, Robert, LL.D.

VOL. LI.

Harker, Alfred, M.A. Hendley, Thomas Holbein, Surgeon Major. Herdman, Professor William Abbott, D.Sc. Hill, Professor M. J. M., M.A. Hinde, George Jennings, Ph.D. Howorth, Henry Hoyle. Hutton, Frederick Wollaston, Capt. R.E. Joly, John, M.A. Jones, Professor John Viriamu, M.A. Kidston, Robert, F.G.S. King, George. Larmor, Joseph, D.Sc. Love, Augustus Edward Hough, M.A. McConnell, James Frederick Parry, Surgeon - Major, F.R.C.P. MacMunn, Charles, M.D. Martin, John Biddulph, M.A.

Gotch, Francis, M.R.C.S.

Matthey, Edward, F.C.S. Miall, Professor Louis C. Newton, Edwin Tully, F.G.S. Notter, James Lane, Surgeon-Lieut.-Col. Oliver, John Ryder, Major-General Peach, Benjamin Neve, F.G.S. Professor Pedler, Alexander, F.C.S. Reade, Thomas Mellard, F.G.S. Roberts, Ralph A., M.A. Rutley, Frank, F.G.S. Sankey, M. H. P. R., Capt. R.E. Saunders, Howard, F.Z.S. Seebohm, Henry, F.L.S. Sherrington, Charles Scott, M.B.

Smith, Rev. Frederick John, M.A. Stebbing, Rev. Thomas Roscoe Rede, M.A. Stevenson, Thomas, M.D. Stirling, Edward C., M.D. Tuke, Daniel Hack, M.D. Ulrich, Professor George Henry Frederic, F.G.S. Veley, Victor Hubert, M.A. Waller, Augustus D., M.D. Waterhouse, James, Colonel. Woodward, Horace Bolingbroke, F.G.S. Worthington, Professor Arthur Mason, M.A. Young, Professor Sydney, D.Sc.

The Right Hon. Spencer Compton Cavendish, Duke of Devonshire, a Member of Her Majesty's Most Honourable Privy Council, whose certificate had been suspended as required by the Statutes, was balloted for and elected a Fellow of the Society.

The following Papers were read:-

I. "Certain Correlated Variations in Crangon vulgaris." By W. F. R. Weldon, M.A., F.R.S., Fellow of St. John's College, Cambridge, Professor of Zoology in University College, London. Received February 11, 1892.

The first successful attempt to find a constant relation between the variations in size exhibited by one organ of an animal body and those occurring in other organs was made some three years ago by Mr. Galton; and in a paper read before the Royal Society ('Roy. Soc. Proc.,' vol. 45, p. 135) he determined this relation between several organs of the human body. In what follows an attempt is made to apply Mr. Galton's method to the measurement of the correlation between four organs of the common shrimp. Before the details of the measurement are discussed, a short summary of the method will be given.

Galton's starting point was the fact that each organ of a given race of men varies about its mean size to an extent and with a frequency indicated by the probability equation $\left(y = \frac{A}{\sqrt{\pi \cdot c}} e^{-x \cdot c^2}\right)$. If two variable organs are known to vary in this way, and if they are so

connected that when the deviation of one variable from its average is known the mean deviation of the second is known, then, evidently, a surface can, from these data, be constructed, showing the relative frequency of occurrence of all possible combinations between the two variables. The changes produced in such a surface by changes in the degree of interdependence of the two variables have been investigated, at Mr. Galton's request, by Mr. J. D. H. Dickson ('Roy. Soc. Proc.,' vol. 40, 1886, p. 63). The results of this investigation, which are of importance for the present purpose, are two:—

(1.) In the population examined, let all those individuals be chosen in which a certain organ, A, differs from its average size by a fixed amount, Y; then, in these individuals, let the deviations of a second organ, B, from its average be measured. The various individuals will exhibit deviations of B equal to x_1, x_2, x_3, \ldots , whose mean may be called x_m . The ratio x_m/Y will be constant for all values of Y.

In the same way, suppose those individuals are chosen in which the organ B has a constant deviation, X; then, in these individuals, y_m , the mean deviation of the organ A, will have the same ratio to X, whatever may be the value of X.

(2.) The ratios x_m/Y and y_m/X are connected by an interesting relation. Let Q_a represent the probable error of distribution of the organ A about its average, and Q_b that of the organ B; then—

$$rac{y_m/X}{x_m/Y} = rac{Q^2_a}{Q^2_b}; \; ext{ or } \qquad rac{x_m/Q_b}{Y/Q_a} = rac{y_m/Q_a}{X/Q_b} = r, ext{ a constant}.$$

So that by taking a fixed deviation of either organ, expressed in terms of its probable error, and by expressing the mean associated deviation of the second organ in terms of its probable error, a ratio may be determined, whose value becomes ± 1 when a change in either organ involves an equal change in the other, and 0 when the two organs are quite independent. This constant, therefore, measures the "degree of correlation" between the two organs.

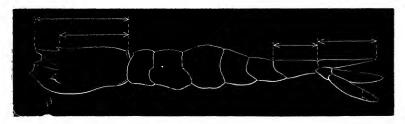
A determination of this constant will now be made in the case of five pairs of organs of the shrimp. In accordance with Mr. Galton's notation, the constant will be denoted by r, the mean size of each organ by M, and the probable error of distribution about the mean by Q.

The organs measured are shown in the woodcut fig. 1; they are:-

- (1.) The total carapace length, measured in a straight line;
- (2.) The length of that portion of the carapace which lies behind the single gastric spine;
- (3.) The length of the sixth abdominal tergum;
- (4.) The length of the telson.

The measurements made were recorded to within 0.05 mm., and

Fig 1.



are expressed in terms of the body length, taken as 1,000. As the average length of the shrimps used was rather over 50 mm, the measurements, in the form in which they are recorded, are accurate only to the nearest unit. The roughness of the edges of the parts measured, and the fact that the animals were preserved in spirit, made any attempt to attain greater accuracy exceedingly difficult.

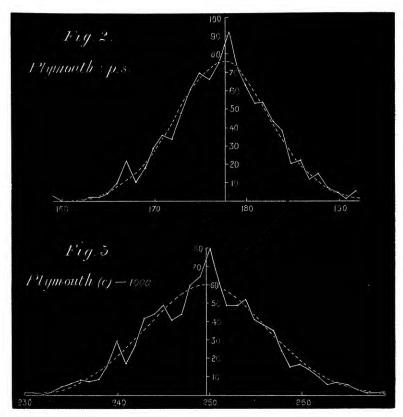
The variations in the length of the four organs here discussed have already been shown to occur with a frequency which agrees very closely with that indicated by the law of probability (cf. 'Roy. Soc. Proc.,' vol. 47, p. 445). The closeness with which the distribution of deviations in the samples used agreed with that indicated by a probability curve may be gathered from the diagrams, figs. 2 and 3, which are fairly typical of the whole series.

1.—Total Length of Carapace and Length of Post-spinous Portion.

The relation between these two parts has been determined in five races of shrimps; of these the sample containing the greatest number of individuals was obtained at the laboratory of the Marine Biological Association at Plymouth.

The mean length of the carapace, in 1000 adult female shrimps from Plymouth, was 249.63 thousandths of the body length; the probable error of distribution about this average was 4.55. These numbers will be denoted in what follows by M_c and Q_c respectively. The mean length of the post-spinous portion (M_{ps}) was 177.53, its probable error (Q_{ps}) being 3.50.

These numbers having been determined, the individuals were sorted into groups, such that the recorded length of the carapace, c, was the same in each group. The mean length of the post-spinous portion, ps, was then determined in each group. The results are entered in the first two columns of Table I. Each pair of entries in the table gives a datum for determining the mean deviation of ps which is associated with a given value of c. If each value obtained in this way be divided by the probable error of the organ to which it belongs, a series of pairs of values will be obtained, from each of which



Figs. 2 and 3.—Curves of frequency of all observed magnitudes of post-spinous lengths (fig. 2) and total carapace lengths (fig. 3) in 1000 adult female shrimps from Plymouth. The dotted curve is a probability curve of the calculated probable error: the continuous curve shows the results of observation. The horizontal scale represents thousandths of the body length: the vertical scale represents tens of individuals.

the value of r can be separately determined. This process has been performed for each of the values entered in the first two columns of Table I, and the results are recorded in the third and fourth columns.

In Table II, the results of a similar process are shown, but the individuals have been sorted into groups in each of which the length of the post-spinous portion of the carapace is constant, and the mean value of the total carapace length has been determined in each group. As before, the immediate results of these determinations are entered in the first two columns, and the deviation of each entry in these columns from its corresponding average, divided by its "probable error," is given in the third and fourth columns.

Table I.—Mean Value of Post-spinous Carapace Length (ps.) for every observed Value of Total Carapace Length (c) in Plymouth. (1000 individuals.)

$$M_c = 249.63$$
; $Q_c = 4.55$

$$M_{ps} = 177.53$$
; $Q_{ps} = 3.50$

$ \begin{array}{c} \text{Length of} \\ c. \end{array} $	$egin{array}{l} ext{Mean} \ ext{associated} \ ext{length}, extit{\it ps}. \end{array}$	$\frac{c-\mathrm{M}_c}{\mathrm{Q}_c}$.	$rac{ps-\mathbf{M}_{ps}}{\mathbf{Q}_{ps}}.$	Length of ps when $r = 0.81$.	Difference observed, $ps.$
Over 260	188 ·41	+2.95	+3 •11	185 ·87	+2:54
260	185 ·41	+2.28	+2.29	184.00	+1.41
259	183 .25	+2.06	+1.63	183 · 38	-0.13
258	$182 \cdot 25$	+1.84	+1.35	182.76	-0:51
257	182 .34	+1.62	+1 37	182.13	+0:21
256	182 •22	+1.40	+1.31	181.51	+0:71
255	181 · 14	+1.18	+1.03	180 .88	+0.26
254	179 • 98	+0.96	+0.70	180.26	-0.28
253	179 •50	+0.74	+0.26	179 .63	-0.13
252	179 · 17	+0.52	+0.47	179.01	+0:16
251	178.68	+0.30	+0.33	178:38	+0:30
250	177 .71	+0.08	+0.05	177.76	-0.05
249	177 ·39	-0.13	-0:05	177:26	+0.10
248	176:64	-0.36	-0.25	176.01	+0.53
247	175:36	-0.58	-0.62	175 .88	-0.52
246	175 · 20	-0.80	-0.67	175 .26	-0.06
245	173 . 56	-1.01	-1.13	174:66	-1.10
244	173 · 31	-1.24	-1/21	174 01	-0.70
243	173 ·33	-1.46	-1.20	173 .38	-0.02
242	172 ·81	-1.68	-1.35	172.76	+0.05
241	171 .30	-1.90	-1.78	172 ·13	+0.83
240	169 .57	$-2 \cdot 12$	-2.27	171 '51	+1:06
Under 240	170 •33	-2.81	-2.20	169 .55	-0.78

The mean value of r, as determined from these two tables, is 0.81; that is to say, if all those individuals be chosen in which the deviation of one of the two organs from its average length is m times the probable error of that organ, the mean deviation from the average size of the other organ in the individuals chosen will be $0.81 \, m$ times its probable error.

If the variations in size occurred in each of these two organs with a frequency equal to that indicated by a probability curve, then every pair of entries in each table should give the same value of r. As the conformity between the observed frequency and that indicated by probability is only approximate, the individual determinations give values which differ slightly from one another; but these variations are on the whole small, as may be shown in three ways.

In fig. 4, all the determinations of r are indicated: the deviation of the organ whose value is fixed in each case is measured parallel to the axis of y, the mean deviations of the associated organ parallel to the axis of x: so that each pair of values in Tables I and II determines

Table II.—Mean Value of Total Carapace Length (c) for every observed Length of Post-spinous Portion (ps) in Plymouth. (1000 individuals.)

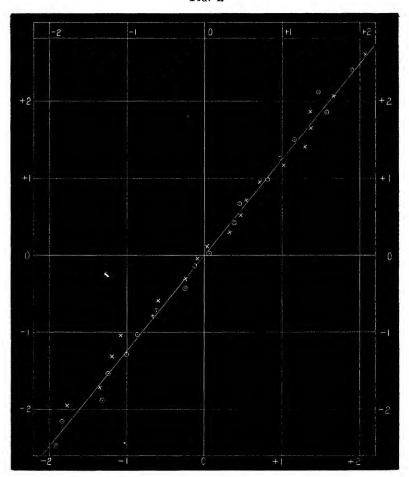
Length of $ps.$	$egin{array}{c} \mathbf{Mean} \\ \mathbf{associated} \\ \mathbf{length} \ \mathbf{of} \ c. \end{array}$	$\frac{ps-\mathbf{M}_{ps}}{\mathbf{Q}_{ps}}$.	$\frac{c-\mathbf{M}_c}{\mathbf{Q}_c}$.	Length of c when $r = 0.81$.	Difference observed,
Over 186	262.11	+3.43	+ 2 · 74	262 • 27	-0.16
186 185	258 ·25 256 ·15	$^{+2\cdot41}_{+2\cdot13}$	+ 1 · 89 + 1 · 43	258·51 257·48	$-0.26 \\ -1.33$
184	256 .84	+1.85	+1.59	256 .45	+0.39
183	254.88	+1.56	+1.12	255 .38	-0.50
182	254.18	+1.28	+1.00	254.36	-0.18
181	253 · 28	+0.99	+0.80	253 .25	+ 0 .03
180	251 .73	+0.71	+0.46	252 .25	-0.52
179	251 ·34	+0.42	+0.38	251.18	+0.16
178	249 .78	+0.13	+0.03	250.77	-0.99
177	249.10	-0.12	-0.15	249 .08	+0.02
17 6	248 · 53	-0.44	-0.24	248.01	+0.52
175	246 · 79	-0.72	-0.62	246 · 97	-0.18
174	245 .73	-1.01	-0.86	245 .90	-0.17
173	245 02	-1.30	-1.01	244 .83	+0.19
172	243 89	-1.58	-1.26	243 .80	+0.09
171	243 · 67	-1.87	-1:31	242 .73	+0.94
170	241 ·28	-2.16	-1.84	241 .76	-0.48
169	241 06	-2.44	-1.91	240 .63	+0.43
Under 169	239 ·88	-3.22	-2.14	237 .75	+2.13

the position of one point in fig. 4. Those points whose position is determined from Table I, taking the carapace length as fixed, are indicated by crosses: those obtained from Table II, taking the post-spinous portion as fixed, are shown by dots surrounded by circles \odot . The straight line drawn across the diagram indicates the ratio x = 0.81 y; and every point, determined from either table, should therefore lie upon it. It will be seen that the points do, in fact, lie fairly closely round the line, though few lie actually upon it.

The accuracy of the assigned value of r may be tested in another way. When the value of r is known, it is evidently possible to calculate, from a known deviation of one organ, the mean associated deviation of the other; and in the fifth columns of Tables I and II the mean length of each dependent organ which should be associated with every observed value of the independent organ has been calculated on the assumption that r=0.81. The length calculated in this way is found to agree very fairly well with the observed length, which is recorded in the second column of each table. The difference between the observed and calculated length of each dependent organ is shown in the sixth column.

A third way of checking the value of r is given by the ratio of the probable error of distribution of one organ to that of the other. If

Fig. 4.



r=0.81, then a deviation of carapace length equal to mQ_c mm. is associated with a mean deviation of the post-spinous portion equal to $0.81 \, mQ_{ps}$ mm.; and from the values of Q_c and Q_{ps} given above, the ratio $0.81 \, mQ_{ps} : mQ_c$ is equal to 0.62. In the same way a deviation of mQ_{ps} mm. in the post-spinous portion of the carapace is associated with a mean deviation of the whole carapace equal to $0.81 \, mQ_c$ mm.; the ratio $0.81 \, mQ_c : mQ_{ps}$ being equal to 1.05.

From the fundamental formula given above on p. 3,

$$\frac{0.62}{1.05} = \frac{Q^2_{ps}}{Q^2_c}$$
.

Now
$$\frac{0.62}{1.05} = 0.590$$
; and $\frac{Q^2_{ps}}{Q^2_{g}} = \frac{(3.5)^2}{(4.55)^2} = 0.591$.

So that the assigned value of r fulfils all the required conditions very fairly well.

Having found a relation between the deviation of carapace lengths and that of post-spinous lengths, which is constant for all magnitudes of either organ in one local race, the question at once arises whether this relation is not a specific character of the shrimp, which is constant in all local races. At the beginning of the inquiry Mr. Galton suggested to me that the relation between the two organs indicated by the value of r was of such a kind that r might be expected to have the same value in all races of the same species, and in some cases in groups of species. A determination of the relation between carapace length and post-spinous length, and of other relations which are to be discussed below, has abundantly confirmed Mr. Galton's prediction.

In order to test the constancy of the relation between them, the variations in total length of carapace, and in length of the post-spinous portion, were measured in samples of adult female shrimps from Helder (North Holland), from Southport, from Sheerness, and from Roscoff (Finistère)—that is, from four places fairly distant from Plymouth, and differing from Plymouth and from each other in climatic conditions, in salinity and other characters of the sea water, and in nature of the sea bottom.* Each of these races was found to differ from the others in the average length of the organs measured, and in the probable error of distribution of each organ about its average; but the relation between the two organs, as measured by the value of r, was very fairly constant throughout.

The details of each determination of r are given in Tables III—X, at the end of the paper; they are constructed on precisely the same plan as that used for Tables I and II, and need not therefore be further explained. The values of r deduced from all the tables are—

In Plymouth	r = 0.81 (1000)	individuals	examined).	
In Southport	0.85 (800	,,	").	
In Roscoff	0.80 (500	,,	,,).	
In Sheerness	0.85 (380	,,	").	
In Helder	0.83 (300	,,	.,).	

The approach to identity between these values is very striking. The differences between them are certainly large; but they are not, as it seems to me, larger than the probable error of each determination. The number of individuals employed, even in the races from Plymouth

^{*} I am glad to express my gratitude to Dr. P. P. C. Hoek, to Professor Delage, and to Messrs. W. Garstang and W. H. Shrubsole, for their kindness in procuring for me these samples.

and Southport, is too small to allow of a satisfactory determination of the second decimal place. The reader, who cares to do so, may satisfy himself of this by taking 0.80 or 0.82 as values of r in the Plymouth tables, when he will find the agreement between calculated and observed values of the associated organ to be only slightly less close than in the existing table.

It may, therefore, be fairly said that the values of r obtained by examining five races of shrimps are not inconsistent with the existence of a constant value in all the races examined—a value lying somewhere between 0.80 and 0.85.

So that if the deviation of total carapace length from its average be expressed in terms of its probable error, and if the deviation of the post-spinous portion be in the same way expressed in terms of its probable error, then, when either organ differs from its average by any constant amount, the mean deviation of the other will be a constant fraction of that amount, the fraction being between 0.80 and 0.85.

A similar approximation to constancy has been found to exist in the relation between three other pairs of organs, determined in the samples from Plymouth and from Southport. These organs are so largely independent that the probable error of the determination is much greater than before; and accordingly the irregularities in the results are much greater than in the previous case. The mean values of r are as follows:—

	Carapace length and tergum vi.	Carapace length and telson.	Telson and tergum vi.
Plymouth	0·06	0·18	-0·11
Southport	0·06	0·14	-9·09

These values are found from the tables at the end of the paper, from which it will be evident that the determination is much less reliable than that first discussed.

So far, we have only investigated the mean deviation of each organ which is associated with a known constant deviation of another organ. But since a fixed deviation of one organ does not as a rule involve a fixed deviation of the second, it becomes necessary to inquire how the values of this second organ are distributed about their mean. Mr. Galton points out that the deviations of each dependent organ are distributed about their mean with a probable error of $Q\sqrt{1-r^2}$. So that in Plymouth, for example, those post-spinous carapace lengths which are associated with any fixed total carapace length should be distributed about their mean, with a probable error of $Q_{ps}\sqrt{1-(0.81)^2}$ or 3.5 0.34, or 2.03 nearly. In the same way, the values of total

carapace length which are associated with a fixed length of the post-spinous portion should be distributed about their mean with a probable error of $4.55\sqrt{0.34}$, or nearly 2.64. Therefore, when the Plymouth shrimps were sorted into groups, such as those used in Table I, such that the carapace length was constant in each, then the post-spinous lengths in each group should have been distributed about the mean given in the table with a probable error of 2.03. The largest of these groups contained only about eighty individuals, so that any accurate determination of this point was out of the question; but a rough estimate of the probable error was made in each group which contained more than forty individuals, and the mean value obtained in this way was 1.96, which is perhaps sufficiently near to 2.03. A similar treatment of the groups in Table II gave 2.59 as the probable error of distribution; and this, again, is not very different from 2.64.

The other samples show a similar rough agreement between the probable error of the dependent organ in each group and that indicated by the value $Q\sqrt{1-r^2}$; but the numbers employed are too small to make a determination of this point worth serious discussion.

It cannot be pretended that the results here given are either sufficiently numerous or sufficiently accurate to serve as a basis for generalisation; but at the same time they seem certainly to suggest a very important conclusion. For if the values of r have really the degree of constancy which has been attributed to them, then by expressing the deviation of every organ examined from its average in terms of the probable error of that organ, the deviation of any one of these organs from its average can be shown to have a definite ratio to the associated deviation of each of the others, which is constant for all the races examined. And since both the organs measured and the samples of shrimps examined were chosen, in the first instance, by chance, any result which holds for all these organs through all these races may be reasonably expected to prove generally true of all organs through the whole species.

That is, the results recorded lead to the hope that, by expressing the deviation of every organ from its average in Mr. Galton's system of units, a series of constants may be determined for any species of animal which will give a numerical measure of the average condition of any number of organs which is associated with a known condition of any one of them. A large series of such specific constants would give an altogether new kind of knowledge of the physiological connexion between the various organs of animals; while a study of those relations which remain constant through large groups of species would give an idea, attainable at present in no other way, of the functional correlations between various organs which have led to the establishment of the great sub-divisions of the animal kingdom.

Table III.—Length of Post-spinous Portion (ps) for every observed Value of Total Carapace Length (c) in Southport. (800 individuals.)

 $M_c = 248.31$; $Q_c = 3.96$.

 $M_{ps} = 180.29$; $Q_{ps} = 3.65$.

$\begin{array}{c} \textbf{Length of} \\ c. \end{array}$	$\begin{array}{c} \textbf{Mean associated} \\ \textbf{length of } ps. \end{array}$	Length of ps when $r = 0.85$.	Difference observed, ps.
over 256 256 255 254 253 252 251 250 249	188 ·70	189 · 43	-0·73
	186 ·50	186 · 30	+0·20
	185 ·37	185 · 36	+0·01
	185 ·48	184 · 61	+0·87
	183 ·65	183 · 83	-0·18
	182 ·84	183 · 08	-0·24
	182 ·37	182 · 33	-0·04
	181 ·43	181 · 58	-0·15
	181 ·27	180 · 83	+0·44
248	179 · 89	180 · 05	$ \begin{array}{c} -0.16 \\ -0.73 \\ -0.86 \\ +0.68 \\ -0.21 \\ +0.39 \\ +1.25 \\ +0.07 \\ +0.41 \end{array} $
247	178 · 57	179 · 30	
246	177 · 69	178 · 55	
245	178 · 45	177 · 77	
244	176 · 81	177 · 02	
243	176 · 66	176 · 27	
242	176 · 76	175 · 49	
241	174 · 81	174 · 74	
under 241	173 · 54	173 · 13	

Table IV.—Value of Total Carapace Length for every observed Length of Post-spinous Portion of Southport. (800 individuals.)

$\begin{array}{c} \text{Length of} \\ \textit{ps.} \end{array}$	Mean associated length of c.	Length of c when $r = 0.85$.	Difference observed, c .
over 189 189 188 187 186 185 184 183 182 181 180 179 178 177	258 · 38 254 · 62 255 · 61 254 · 93 252 · 92 252 · 43 250 · 67 250 · 31 249 · 18 247 · 16 247 · 28 247 · 91 244 · 33 243 · 51 243 · 42	258 · 38 256 · 36 255 · 42 254 · 51 253 · 57 252 · 66 251 · 75 250 · 80 249 · 89 248 · 95 248 · 95 248 · 91 245 · 28 244 · 33 243 · 33	0 · 00 -1 · 74 +0 · 19 +0 · 42 -0 · 75 -0 · 23 -0 · 85 -0 · 13 +0 · 42 +0 · 23 -0 · 88 +0 · 15 +1 · 00 -0 · 95 -0 · 82 +0 · 09
174 173 172 under 172	$241 \cdot 20$ $241 \cdot 48$ $241 \cdot 16$ $238 \cdot 78$	242·48 241·57 240·66 240·05	-1.28 -0.09 $+0.50$ -1.27

Table V.—Mean Length of Post-spinous Portion for every observed Value of Total Carapace Length in Roscoff. (500 individuals).

 $M_c = 251.51$; $Q_c = 3.32$. $M_{ps} = 178.00$; $Q_{ps} = 3.02$.

	Mean associated length of ps.	Length of ps when $r = 0.80$.	Difference observed, ps.
over 257 257 256 255 254 253 252 251 250 249 248 247 246	184 · 54 181 · 70 181 · 67 180 · 88 179 · 89 178 · 50 177 · 89 178 · 02 176 · 75 176 · 75 174 · 68 173 · 54	184 · 75 181 · 99 181 · 27 180 · 54 179 · 81 179 · 18 178 · 36 177 · 64 176 · 91 176 · 45 175 · 46 174 · 71 173 · 98	-0·21 -0·29 +0·40 +0·34 +0·08 -0·32 -0·47 +0·38 +0·61 +0·30 +1·06 -0·03 -0·44
under 246	170 .06	169 • 94	+0.08

Table VI.—Mean Value of Total Carapace Length for every observed Value of Post-spinous Portion. (Roscoff: 500 individuals.)

Length of ps.	$egin{array}{c} \mathbf{M_{ean}} \ \mathbf{associated} \ \mathbf{length}, \ oldsymbol{c}. \end{array}$	Length of c when $r = 0.80$.	Difference observed, c .
Over 184	260 · 34	260 ·98	-0 ·64
184	257 · 45	256 ·80	+0 ·65
183	256 · 28	255 ·70	+0 ·58
182	254 · 57	255 ·02	-0 ·45
181	254 · 69	254 ·14	+0 ·45
180 179 178 177 176 175	253 · 69 252 · 29 251 · 09 250 · 02 250 · 07 248 · 82 246 · 92	253 · 26 252 · 39 251 · 51 250 · 63 249 · 76 248 · 88 248 · 00	+ 0·48 - 0·10 - 0·42 - 0·61 + 0·31 - 0·06 - 1·08
173	245 ·94	247 · 11	-1·17
Under 173	240 ·76	242 · 33	-1·57

Table VII.—Mean Post-spinous Length for every observed Value of Total Carapace Length. (Sheerness: 380 individuals.)

 $\mathbf{M}_{c} = 247 \cdot 33 \, ; \, \, \mathbf{Q}_{c} = 3 \cdot 29 \text{.} \quad \mathbf{M}_{ps} = 179 \cdot 68 \, ; \, \, \mathbf{Q}_{ps} = 2 \cdot 91 \text{.}$

Length of c .	Mean associated length of ps.	Value of ps when $r = 0.85$.	Difference observed, ps.
Over 251	184 · 52	184 · 94	$ \begin{array}{r} -0.42 \\ +3.01 \\ -0.55 \\ -0.59 \\ +0.93 \\ +0.41 \\ +0.07 \\ -0.02 \\ -0.53 \\ +0.78 \\ +1.15 \end{array} $
251	185 · 38	182 · 37	
250	181 · 07	181 · 62	
249	180 · 48	181 · 07	
248	181 · 19	180 · 26	
247	179 · 03	179 · 44	
246	178 · 69	178 · 62	
245	177 · 96	177 · 98	
244	176 · 73	177 · 26	
243	177 · 29	176 · 51	
Under 243	175 · 53	174 · 38	

Table VIII.—Mean Carapace Length for every observed Value of the Post-spinous Portion. (Sheerness: 380 individuals.)

Length of ps.	Mean associated length of c.	Length of c when $r = 0.85$.	Difference observed, c.
Over 183 183 182 181 180 179 178 177. 176. Under 176	253·55 249·29 248·50 249·21 247·70 246·06 245·93 246·29 243·00 242·75	254·97 250·56 249·59 248·60 247·64 246·68 245·69 244·73 243·76 240·68	$\begin{array}{c} -1 \cdot 42 \\ -0 \cdot 67 \\ -1 \cdot 09 \\ +0 \cdot 61 \\ +0 \cdot 04 \\ -0 \cdot 62 \\ +0 \cdot 24 \\ +1 \cdot 56 \\ -0 \cdot 76 \\ +2 \cdot 07 \end{array}$

Table IX.—Mean Length of Post-spinous Portion for every observed Value of Total Carapace Length. (Helder: 300 individuals.)

$\mathbf{M}_c =$	251.38;	$Q_c =$	4.36.	$\mathbf{M}_{ps} =$	181.67;	$Q_{ps} =$	4.02.
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Length of c.	Mean associated length of ps .	Length of ps when $r = 0.83$.	Difference observed, ps .
Over 256	187.98	188 •28	-0.30
256	183 ·85	185 •21	-1.36
255	184 .09	184 · 44	-0.36
254	185.08	183 .67	+1.41
253	182.56	182 .90	-0:34
252	181 • 48	182 ·13	-0.65
251	179.83	181 · 64	-1.81
250	179 ·87	180.60	-0.73
249	179 · 42	179 ·83	-0.41
248	178 · 37	178 .93	-0.56
Under 248	176 · 11	176 • 19	+0.08

Table X.—Mean Value of Total Carapace Length for every observed Length of Post-spinous Portion. (Helder: 300 individuals.)

Length of ps.	Mean associated length of c.	Length of c when $r = 0.83$.	Difference observed, c.
Over 187	259 •97	260 • 57	-0.60
187	254.83	256 · 19	-1.36
186	255 .92	255 · 18	+0.74
185	255.05	254 ·38	+0.67
184	252 · 56	253 · 48	-0.92
183	253 .28	252 .57	+0.71
182	251.78	251 .66	+0.12
181	250.63	250.77	-0.14
180	249 • 92	249 .86	+0.06
179	247 :31	248.99	-1.68
178	248 • 95	248.09	+0.86
Under 178	247.00	243 ·89	+3.11

Table XI.—Mean Length of Sixth Abdominal Tergum for each Value of Total Carapace Length. (Plymouth: 1000 individuals.)

 $M_{\it c} = 249 \cdot 63 \ ; \ Q_{\it c} = 4 \cdot 55. \qquad \qquad M_{\rm Tvi} = 145 \cdot 71 \ ; \ Q_{\rm Tvi} = 2 \cdot 82.$

Length of c .	Mean associated length of Tvi.	Tvi when $r = 0.09$.	Difference observed, Tvi.
257	146 • 91	146.12	+0.79
256	146 ·46	146.06	+0.40
255	145 .95	146.00	-0.05
254	145 ·10	145.95	-0.85
253	145.35	145 ·89	-0.54
252	146 .76	145 · 84	+0.92
251	145 • 93	145 .78	+0.15
250	145 .70	145.73	-0.03
249	145 .47	145 .68	-0.21
248	146.02	145 .62	+0.40
247	145 • 44	145 .56	-0.12
246	146 :30	145 •51	+0.79
245	145 .96	145 •46	+0.20
244	144.22	145 .40	-1.18
243	146 ·17	145 · 34	+0.83
242	145 .68	145 • 29	+0.39

Table XII.—Value of Total Carapace Length for observed Lengths of Sixth Abdominal Tergum. (Plymouth: 1000 individuals.)

Length of Tvi.	Mean associated length of c .	Length of c when $r = 0.09$.	Difference observed, c .
151 150 149 148 147 146 145 144 143 142 141	250 · 26 250 · 00 249 · 79 250 · 42 249 · 46 249 · 19 248 · 08 249 · 24 249 · 84 248 · 72 248 · 55 249 · 06	250·40 250·25 250·11 249·99 249·81 249·67 249·53 249·38 249·24 249·09 248·95 248·81	$\begin{array}{c} -0.14 \\ -0.25 \\ -0.32 \\ +0.43 \\ -0.35 \\ +0.48 \\ -1.45 \\ -0.14 \\ +0.60 \\ -0.37 \\ -0.40 \\ +0.25 \end{array}$

Table XIII.—Mean Length of Sixth Abdominal Tergum for observed Values of Total Carapace Length. (Southport: 800 individuals.)

$M_c = 248.31$;	$Q_c = 3.96.$	$M_{Tvi} =$	143.56;	$Q_{Tvi} =$	3.20.
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Length of c .	Mean associated length of Tvi.	Length of Tvi when $r = 0.05$.	Difference observed, Tvi.
259	145 · 57	143 99	+1:58
258	140.75	143 . 95	-3.20
257	144 . 25	143 .91	+0.34
256	142.74	143.87	-1.13
255	142 .84	143 · 83	-0.99
$\bf 254$	143 .92	143.79	+0.13
253	143 · 41	143.75	-0.34
252	144.54	143.71	+0.83
251	144 97	143 · 67	+1.30
250	143 .26	143 · 63	-0.37
249	143 - 23	143 · 59	-0.36
248	143.56	143 · 55	+0.01
247	143 · 67	143.51	+0.16
246	143.90	143 · 47	+0.43
245	144 · 16	143 ·43	+0.73
244	142 · 57	143 · 39	-0.82
243	143 .88	143 · 35	+0.33
242	142 · 48	143 · 30	-0.82
241	143 '41	143 · 26	+0.15
240	142 ·33	143 · 22	-0.89

Table XIV.—Mean Value of Total Carapace Length for observed Lengths of Sixth Abdominal Tergum. (Southport: 800 individuals.)

Length of Tvi.	Mean associated length of c.	Length of c when $r = 0.05$.	Difference observed, c.
150	249 ·15	248 71	+0.34
149	$248 \cdot 74$	248 ·65	+0.09
1.48	$247 \cdot 44$	248 59	-1.05
147	247.53	248.52	-0.99
146	$249 \cdot 47$	248 • 46	+1.01
145	248.71	248.38	+0.33
144	$247 \cdot 40$	248 · 34	-0.94
143	247.76	248 · 28	-0.52
142	$247 \cdot 35$	248 · 21	-0.86
141	$247 \cdot 26$	248 · 14	-0.88
140	249.14	248.09	+1.05
139	245.51	248 · 03	-2.52
138	248 .69	247 .95	+0.74
137	246.81	247 .90	-1.09

Table XV.—Mean Length of Telson for observed Values of Total Carapace Length. (Plymouth: 1000 individuals.)

 ${
m M}_c = 249^{\circ}63 \; ; \; {
m Q}_c = 4^{\circ}55. \quad {
m M}_{
m Te} = 193^{\circ}08 \; ; \; {
m Q}_{
m Te} = 4^{\circ}56.$

Length of c .	Mean associated length of Te.	Length of Te when $r = 0.18$.	Difference observed, Te.
258	193 · 79 194 · 92	194 · 59 194 · 41	-0.80 +0.51
$257 \\ 256 \\ 255$	194 92 195 · 78 196 · 08	194 41 194 · 23 194 · 05	$+1.55 \\ +2.03$
$254 \\ 253$	196 · 32 193 · 87	193 · 87 193 · 69	$+8.45 \\ +0.18$
$\begin{array}{c} 252 \\ 251 \end{array}$	194 ·53 194 ·48	193 · 51 193 · 32	+1.02 +1.16
$\begin{array}{c} 250 \\ 249 \end{array}$	$192 \cdot 97$ $193 \cdot 43$	193 · 15 192 · 95	-0.18 + 0.48
$\begin{array}{c} 248 \\ 247 \end{array}$	194 · 58 192 · 00	192·79 192·51	$+1.78 \\ -0.51$
$246 \\ 245 \\ 241$	192 · 32 191 · 91	192 · 33 192 · 15	-0.01 -0.24
$\begin{array}{c} 244 \\ 243 \end{array}$	192 ·28 190 ·29	192 · 07 191 · 89	$+0.21 \\ -1.60$

Table XVI.—Value of Total Carapace Length for observed Length of Telson. (Plymouth: 1000 individuals.)

 $M_c = 249.63$; $Q_c = 4.55$.

$$M_{Te} = 193.08$$
; $Q_{Te} = 4.56$.

Length of Te.	Mean associated length of c.	c when $r = 0.18$.	Difference observed, c.
199 198 197 196 195 194 193 192 191 190 189	259·18 244·25 250·54 249·39 249·37 248·81 248·30 249·27 247·82 249·36 248·74	250·69 250·51 250·33 250·15 249·97 249·79 249·61 249·43 249·25 249·07 248·90 248·71	+8.49 -6.26 $+0.21$ -0.76 -0.60 -0.98 -1.31 -0.16 -1.43 $+0.29$ -0.16 $+0.20$

Table XVII.—Mean Length of Telson for observed Values of Total Carapace Length. (Southport: 800 individuals.)

 $M_c = 248.31$; $Q_c = 3.96$.

 $M_{Te} = 195.48$; $Q_{Te} = 3.71$.

Length of c.	Mean associated length of Te.	Length of Te when $r = 0.14$.	Difference observed, Te.
256	196 · 25	196 · 49	-0:24
255	196.00	196 . 36	-0.36
254	196 ·81	196 .23	+0.28
253	196.75	196.09	+0.66
252	195.70	195 .06	-0.26
251	195 · 48	195 ·83	-0.35
250	194.81	195 .70	-0.89
249	194.05	195 · 57	-1.52
248	195 .95	195 44	+0.51
247	194.94	195 - 31	-0.37
246	195 - 22	195 ·18	+0.04
245	195 .28	195 05	+0.23
244	195 · 29	194 · 92	+0.37
243	194.92	194.79	+0.13
242	193 -93	194.65	-0.72

Table XVIII.—Mean Value of Total Carapace Length for observed Lengths of Telson. (Southport: 800 individuals.)

Length of Te.	Mean associated length of c.	Length of c when $r = 0.14$.	Difference observed, c .
202	247 .04	249 ·27	-2.23
201	250 . 78	249 • 13	+1.65
200	247 ·11	248 .98	-1.87
199	248 • 21	248 ·83	-0.62
198	249 · 23	248 .68	+0.55
197	247 46	248 · 53	-1.07
196	248 .05	248.39	-0.34
195	247 ·81	248 · 26	-0.45
194	247 15	248 12	-0.97
193	248 • 42	247 · 97	+0.45
192	248 · 61	247 .82	+0.79
191	248 · 31	247 67	+0.64
190	244 ·81	247 . 52	-2.71
189	247 · 46	247 37	+0.09
188	$246 \cdot 38$	247 · 22	-0.84
187	248 .04	247.07	+0.97

Table XIX.—Mean Length of Sixth Abdominal Tergum for observed Lengths of Telson. (Plymouth: 1000 individuals.)

 $M_{\mathrm{Tvi}} = 145.71$; $\,Q_{\mathrm{Tvi}} = 2.82.$

 $M_{Te}=193.08$; $\,Q_{Te}=4.55.$

Length of Te.	Mean associated length of Tvi.	Length of Tvi when $r = -0.11$.	Difference observed, Tvi.
200 199	145·69 144·40	145 · 23 145 · 30	+ 0 · 46 - 0 · 90
198 197 196	144.58 143.54 145.87	$145 \cdot 37$ $145 \cdot 44$ $145 \cdot 51$	-0.79 -1.90 $+0.36$
195 194	$145 \cdot 29 \\ 145 \cdot 48$	145 · 58 145 · 65	$-0.29 \\ -0.17$
193 192 191	$ \begin{array}{c c} 144 \cdot 91 \\ 145 \cdot 17 \\ 147 \cdot 50 \end{array} $	$egin{array}{ccc} 145 \cdot 72 \\ 145 \cdot 78 \\ 145 \cdot 85 \\ \end{array}$	-0.81 -0.61 $+1.65$
190 189 188	$147 \cdot 24$ $145 \cdot 83$ $142 \cdot 54$	145.93 146.00 146.07	$\begin{array}{c} +1.31 \\ -0.17 \\ -3.53 \end{array}$
187	145 ·53	146 14	-0.61

Table XX.—Mean Length of Telson for observed Lengths of Sixth Abdominal Tergum. (Plymouth: 1000 individuals.)

Length of Tvi.	Mean associated length of Te.	Te when $r = -0.11$.	Difference observed, Te.
150 149 148 147 146 145 144 143 142 141	193 ·92 191 ·60 193 ·42 193 ·22 192 ·99 193 ·94 193 ·13 192 ·66 193 ·24 191 ·02	193 ·70 193 ·56 193 ·41 193 ·27 193 ·12 192 ·98 192 ·88 192 ·69 192 ·55 192 ·40 192 ·26	+ 0 · 22 - 1 · 96 + 0 · 01 - 0 · 05 - 0 · 13 + 0 · 96 + 0 · 30 - 0 · 03 + 0 · 69 + 0 · 84 - 1 · 24

Table XXI.—Mean Length of Sixth Abdominal Tergum for observed Lengths of Telson. (Southport: 800 individuals.)

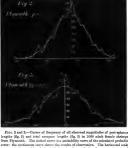
 $M_{Te} = 195 {\cdot} 48 \, ; \, \, Q_{Te} = 3 {\cdot} 71. \quad M_{Tvi} = 143 {\cdot} 56 \, ; \, \, Q_{Tvi} = 3 {\cdot} 20.$

Length of Te.	Mean associated length of Tvi.	Length of Tvi when $r = -0.09$.	Difference observed, Tvi.
204	143 · 78	142.68	+0.90
203 202	$144.60 \\ 144.96$	$142.96 \\ 143.04$	$^{+1\cdot 64}_{+1\cdot 92}$
201	142 ·41	143.12	-0.71
$\begin{array}{c} 200 \\ 199 \end{array}$	142.02	$143 \cdot 20 \\ 143 \cdot 28$	$-1.18 \\ -0.92$
198	142.36 142.95	143 26	-0.41
197	143 .54	143 · 44	+ 0 ·10
$\frac{196}{195}$	$141.23 \\ 143.88$	143 · 52 143 · 60	-2·29 +0·28
194	144.35	143 .68	+1.27
$\frac{193}{192}$	143 ·95 144 ·18	143:76 143:84	+0·19 +0·34
192	142 91	144 • 00	-1 09
190	140 .03	144.08	-4.05
189	142 ·36	144 ·16	-1.80

Table XXII.—Mean Length of Telson for observed Lengths of Sixth Abdominal Tergum. (Southport: 800 individuals.)

Length of Tvi.	Mean associated length of Te.	Length of Te when $r = -0.09$.	Difference observed, Te.
150	194 · 14	194.81	-0.67
149	193 .72	194.92	-1.20
148	196 · 64	195.02	+1.62
147	195 · 31	195 ·12	+0.19
146	196.56	195 .23	+1.33
145	195 .07	195 · 33	-0.26
144	195 .97	195 · 44	+0.23
143	194.10	195 • 54	-1.44
142	195 .77	195.64	+0.13
141	193 .98	195.75	-1.77
140	194 · 40	195 .85	-1.45
139	195 ·34	195 · 95	-0.61
138	195.59	196.06	-0.47
137	196 · 16	196 · 16	0.00





epresents thousandths of the body length; the vertical scale represents tens of

